

Use of Simulation and Real Options Applications in Natural Resources/Energy

Real Options Valuation in the Modern Economy
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Presentation Outline

- ◆ **Introduction and overview of Monte Carlo (MC) simulation applied to finance.**
 - MC concept. Efficient sampling (MC x quasi-MC).
 - Real x Risk-Neutral simulation of stochastic processes.
 - Equations for simulation of continuous-time stochastic processes: exact and approximate discretizations.
 - MC simulation for European and for American options.
- ◆ **Applications of MC simulation in real options.**
 - Flex-Fuel plant (input flexibility: fuel oil x coal).
 - The biodiesel project.
 - The Bolivia-Brazil gas pipeline tariff case.
 - Learning options + timing options: combining technical and market uncertainties in oilfield development.

Monte Carlo Simulation: Introduction

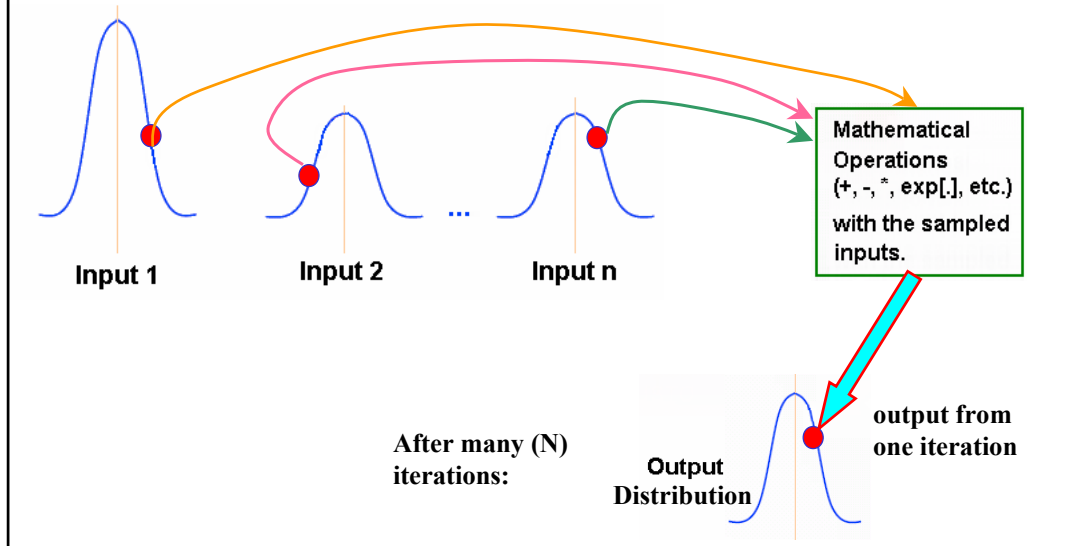
- ◆ Monte Carlo simulation is an increasingly popular method to value complex derivatives, including real options.
 - The MC method solves a problem by simulating directly the physical process, so that it is not necessary to write down the differential equations that describe the system behavior.
 - It is a very **flexible tool** to handle many specific details of real life problems, including many boundary and other system restrictions and **many source of uncertainties**.
 - In short, it is the antidote to both the *curse of dimensionality* and the *curse of modeling* that plague complex real life problems.
- ◆ The MC idea and name are attributed to S. Ulam and N. Metropolis, respectively, in the Manhattan Project at Los Alamos during the World War II times.
 - The first Monte Carlo paper, named "The Monte Carlo Method", by Metropolis & Ulam, was published in 1949 in the *Journal of the American Statistical Association*.

Monte Carlo Simulation at Work

- ◆ MC method consists basically of:
 - (a) specify the **input variable distributions** (including sequence of distributions along the time, i.e., stochastic processes) and eventual correlations/dependences;
 - (b) **sample** these input distributions;
 - (c) do **mathematical operations** with the sampled inputs (+, −, *, ^, /, exp[.], etc.) **to calculate the output** from these samples;
 - (d) **repeat** the previous steps **N times**, generating N outputs; and
 - (e) calculate the mean and other probabilistic properties from the resulting **output distribution**.
- ◆ The figure in the next slide illustrates this procedure.

Monte Carlo Simulation at Work

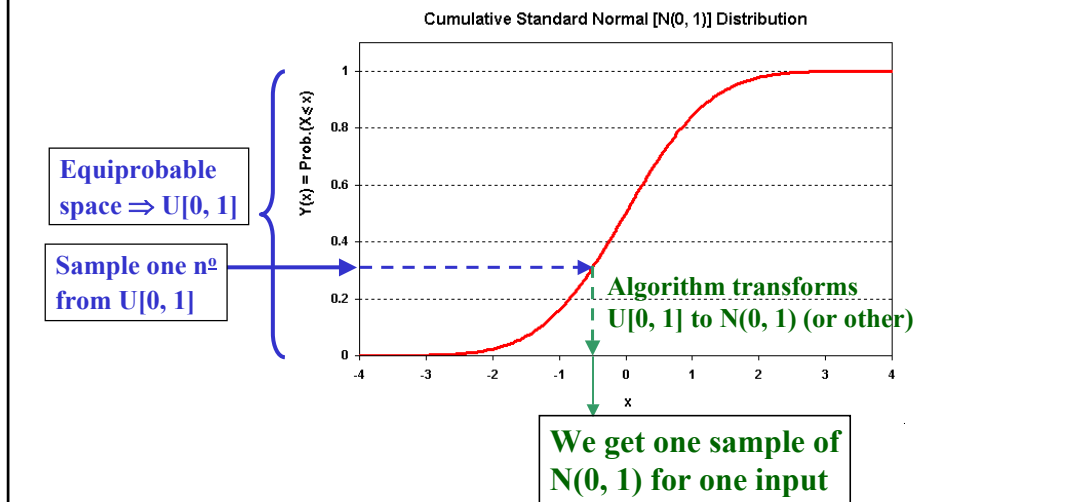
◆ The MC method is illustrated with the figure below:



MC Simulation: Sampling Process

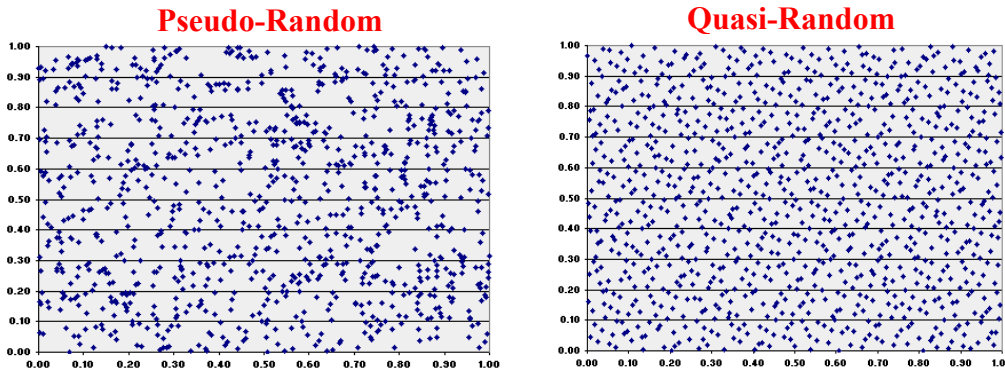
◆ The simulation numerical quality depends on the sampling process. Figure below (standard Normal) shows that in general we need only the uniform distribution for the interval $[0, 1]$.

- Algorithms transform the $U[0, 1]$ sample to other distribution sample.



Generating U[0, 1]: Pseudo x Quasi-Random

- ◆ The simulation numerical accuracy depends on the quality of U[0, 1] generation. In order to generate U[0, 1] the traditional approach is the *pseudo-random* one, e.g., Excel function Rand().
 - A better approach is to generate **quasi-random numbers** (sequences of *low discrepancy*). The figure below compares the two approaches for the bi-dimensional uniform. Note that quasi-random case shows a more evenly dispersed points (less clustered than pseudo-random).



Real x Risk-Neutral Stochastic Processes

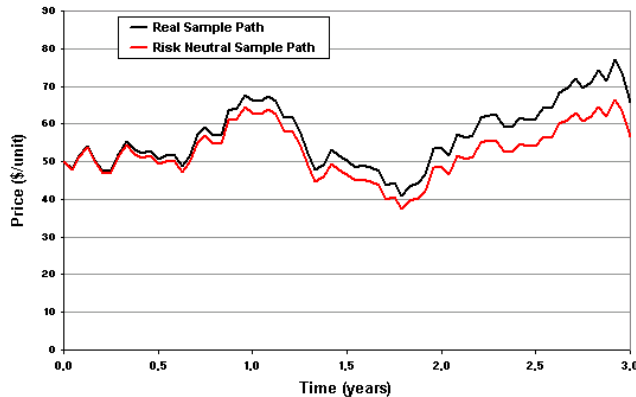
- ◆ We can simulate either the real stochastic process or the risk-neutral stochastic process. The difference is a risk-premium π subtraction from the real drift α .
 - Risk-neutral simulation is used for derivatives pricing because we don't know (or is hard) the derivative's risk-adjusted discount rate.
 - So we penalize the drift and so the distribution (lower mean), a *martingale change of measure*, in order to use the risk-free discount rate for the derivative.
 - Real drift = α \Rightarrow **Risk-neutral drift = $\alpha - \pi = r - \delta$**
- ◆ For the geometric Brownian motion (GBM), used in Black-Scholes-Merton, the real and risk-neutral GBMs are:

$$\frac{dP}{P} = \alpha dt + \sigma dz \quad \Rightarrow \quad \text{Real GBM.}$$

$$\frac{dP}{P} = (r - \delta) dt + \sigma dz' \quad \Rightarrow \quad \text{Risk-Neutral GBM.}$$

Real x Risk-Neutral Stochastic Processes

- ◆ A typical sample-paths for both real and risk-neutral GBMs (with the same stochastic shocks) is showed: the difference is π .



- ◆ While risk-neutral simulation is used to price derivatives, real simulation is useful for *planning purposes* (e.g., if wait and see is optimal, what is the probability of option exercise?) and for *risk-analysis* (e.g., value-at-risk estimation) & hedging.

Equations for Stochastic Process Simulation

- ◆ Some stochastic processes (not all) admit *exact discretization*, i.e., numerical precision inderpends of the time-step length.
 - This is particularly useful for real options, because we work with long time to expiration, e.g., we can use $\Delta t = 1$ year without losing precision.
- ◆ The exact discretization equations to simulate both the **real** and **risk-neutral** geometric Brownian motions are, respectively:

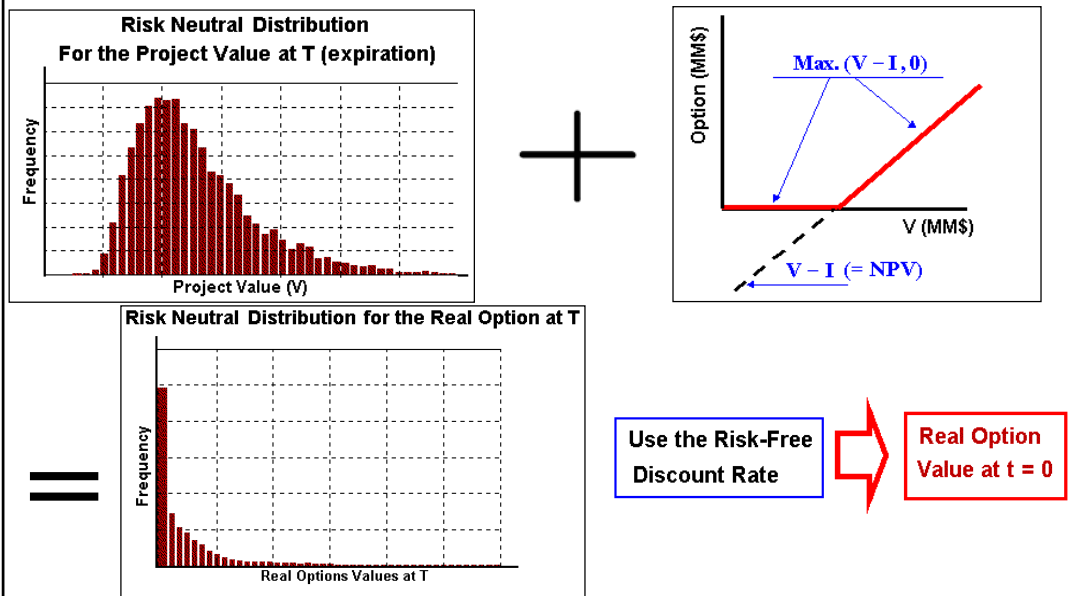
$$P_t = P_0 \exp\{(\alpha - 0.5 \sigma^2) \Delta t + \sigma N(0, 1) \sqrt{\Delta t}\}$$

$$P_t = P_0 \exp\{(r - \delta - 0.5 \sigma^2) \Delta t + \sigma N(0, 1) \sqrt{\Delta t}\}$$

- The difference is the drift. Sampling $N(0, 1)$ n times, we get n outputs P_t .
- ◆ Stochastic processes with exact discretizations include mean-reversion. See: www.puc-rio.br/marco.ind/sim_stoc_proc.html
- ◆ We can simulate the entire GBM path or only at the expiration (European options). The European options can be calculate by simulation and compared with the Black-Scholes analytic result.

European Call Valuation by Simulation

- ◆ If the underlying asset V is the operating project and I is the exercise price (investment), the visual equation for European real option is:



European & American Real Options

- ◆ Valuation by simulation of European style real options is very easy because the decision rule is known, e.g., $\text{Max}[V - I, 0] @ T$.
- ◆ American real options are much harder to solve by simulation because the optimal decision rule is much more complex:
 - The optimal decision rule for American option is the threshold curve, which is calculated *backwards* whereas simulation is *forward* looking.
- ◆ However, if we know in advance the threshold curve or if we combine the MC simulation with one optimization method, we can evaluate American real options by simulation.
 - Since the nineties, there is a growing literature on new methods to evaluate American options by simulation. The best known paper is Longstaff & Schwartz (2001), but there are more than 20 methods.
- ◆ Knowing the threshold curve *after* a learning option exercise, we can evaluate complex learning + timing options with MC.
 - We'll see later an oilfield development case with learning (Dias, 2002) combining technical and market uncertainties with MC simulation.

European Real Options by Simulation

- ◆ There are many practical problems that we can apply the European option valuation by Monte Carlo simulation, mainly sequence of European real options (e.g., calls on a basket of assets).
 - This is best way to value projects with **flexible inputs and/or flexible outputs**, because at specific decision dates (ex.: every month) the firm has to decide the best mix of inputs and outputs for the next operational period (to maximize the payoff, e.g., for the next month).
 - We'll see some real life cases. The idea is to simulate the risk-neutral stochastic processes for the inputs and outputs prices, which are not necessarily GBMs (e.g., could be mean-reversion with jumps).
 - In addition, the exercise payoff function can be very complex, with many real life details (e.g., one input is not available in the first year or in certain months; a minimum quantity of one input must be used due to a contract commitment, etc.).
 - MC simulation plugged into a spreadsheet is very flexible to handle multiple/complex stochastic processes and complex payoff functions.

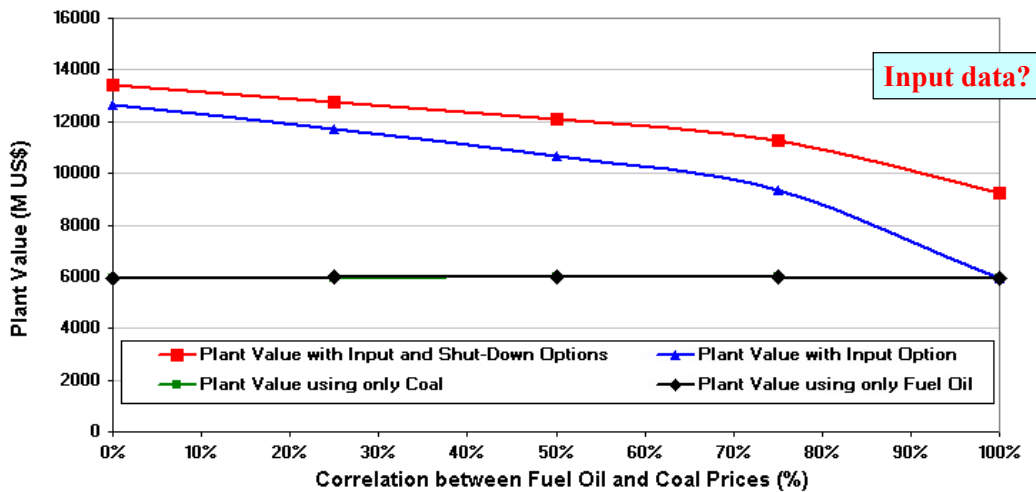
Flex-Fuel Plant with & without Shut-Down Option

- ◆ One firm is going to invest in a energy consuming plant. There are three energy technology alternatives:
 - Plant using only oil fuel; plant using only coal; and flex-fuel plant, i.e., plant with (costless) input flexibility (oil or coal).
 - We'll see also the flex-fuel plant with costless *shut-down option*.
- ◆ What are the plant values in each case considering that oil fuel and coal follow correlated mean-reversion processes?
 - The answer gives an idea of the maximum value that a firm is willing to pay for the (more expensive) flex-fuel technology.
 - Positive correlation decreases the option value, but it is necessary a (unlikely) very high correlation for the input option be negligible.
- ◆ What is the effect of the costless shut-down option?
 - This option can be very important. There are contract implications.
- ◆ MC simulation answers easily these questions.
 - This is a *sequence of European options* (choose the maximum payoff at each operational decision date). The next slide shows an example.

Flex-Fuel Plant, Correlation & Flexibility Value

◆ The chart shows a numerical example with mean-reversion for both oil fuel and coal, for different correlations.

- The chart numerical values were obtained with MC simulation.
- Plant values using only one input (without options) are ~ the same.



Real Life Application: Biodiesel Project

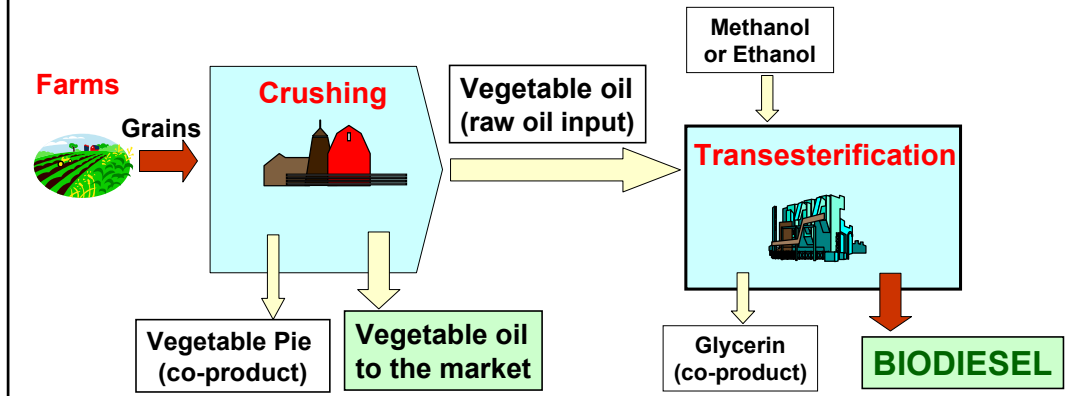
◆ Biodiesel fuel for diesel engines has low emission advantage and is produced from vegetable oil or animal fat by the chemical process of transesterification with alcohols.

- Commercial biodiesel production in US started in late 1990's.
 - Biodiesel as fuel additive, will be obligatory in Brazil in 2008.
- ◆ We are considering only multi-vegetable biodiesel plants.
- So, there is *input flexibility* to choose the vegetable that maximize the project value. Real options is the natural tool to evaluate this.
 - Some Brazilian vegetable considered were soybean, cotton, castorbean, pinion (*jatropha curcas*), uricury syagrus palm, etc.
 - In addition, there is input reagent flexibility: methanol or ethanol.
 - The vegetables price (and their oils) and the alcohols are commodities and oscillate in the market.
 - ➔ We use stochastic processes to model these uncertain prices.

Biodiesel Plant, Inputs and Outputs

◆ A biodiesel plant has two main units:

- The **crushing** unit, the vegetable grain is crushed generating raw oil and residue (pie). Raw (vegetable) oil is the main revenue.
- The **transesterification** unit, that uses raw vegetable oil (cost) and reagent (alcohol), generating biodiesel plus residuals .
- The figure below shows the biodiesel plant and its inputs/outputs.



Biodiesel Project: The Value of Input Flexibility

◆ Petrobras biodiesel business format: owner of *both units*, (crushing and transesterification). Why crushing unit?

- In order to guarantee the raw oil quality; and
- In order to *capture the flexibility* (real option) *value* in choosing the vegetable grain input.

◆ This flexibility is modeled as a *sequence of European call options on maximum of several risky assets*:

- At each period the biodiesel plant choose the vegetable(s) and reagent combination that maximizes the profit in that period.
- We performed Monte Carlo simulations for the stochastic processes of the input prices (several grains, vegetable raw oils, methanol, ethanol) and the output prices (biodiesel = diesel, residues, and vegetable oils to the market).
 - Difficulties to estimate some stochastic process parameters (lack of data).
- The flexibility (real options value) added a significant and decisive value for biodiesel project economic feasibility. **Jump to conclusions?**

Bolivia-Brazil Gas Pipeline Tariff Case

- ◆ In 2000/2001, arbitrated by ANP, took place a dispute between TBG (controlled by Petrobras) and entrants (BG and Enersil) on the Bolivia-Brazil gas pipeline tariff.
 - They wanted pay a tariff value equal to the *ship-or-pay* tariff, but without the ship-or-pay obligation!
 - ➔ The entrants had more flexibility: if gas demand drops they can leave the pipeline without paying ship-or-pay tariff. If demand rises, they profit by signing shipments contracts.
 - ➔ We argued that the entrants flexibility has value so that their tariff shall be higher (option premium) than ship-or-pay one.
- ➔ What is the fair flexibility premium for this tariff?
- ◆ The answer is: given the ship-or-pay tariff, a fair “flexible” tariff makes a firm indifferent between paying the cheaper ship-or-pay or paying the more expensive tariff but with flexibility to leave (ship-or-pay plus a positive premium).

Bolivia-Brazil Gas Pipeline Tariff Case

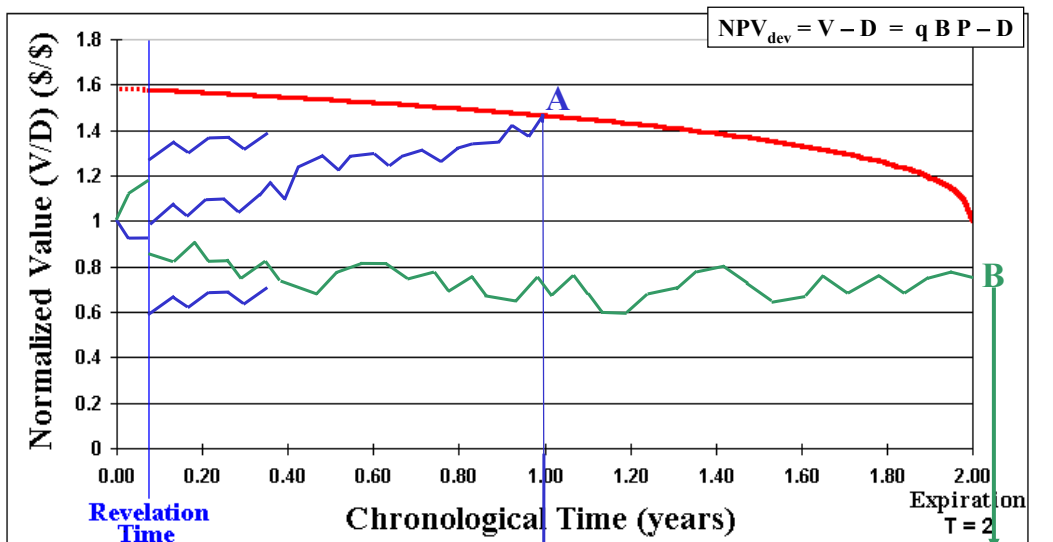
- ◆ To estimate the fair flexibility premium for the non-ship-or-pay tariff, we performed a MC simulation of the Brazilian gas market for the term in dispute (3 y.)
 - By using time-series, we saw that *geometric Brownian motion* was a good model for the gas demand in the next three years.
 - We estimated the GBM parameters (mainly the volatility).
 - We set standard contracts of 100,000 m³/d, with one year term. Entrants exercise the options to sign contracts in case of *excess of demand* (demand higher than the current contracted volume). If demand drops, these contracts are not renewed.
 - We simulated the competition about the expected excess of demand *share* that could be captured by entrants and others.
 - We calculated the present values for a firm using ship-or-pay and flexible tariff. The fair tariff equals these present values.
 - Result: We estimated a 20% premium. ANP determined 11%.

Learning + Timing Options in Oilfield Development

- ◆ This case was presented in Dias (2002):
 - One oilfield has remaining technical uncertainty about the reserve volume (B) and the reserve productivity (quality q).
 - In addition, long-term oil prices (P) and the development investment (D) are uncertain and follow correlated GBMs.
 - The development exercise payoff is $NPV = V - D = q B P - D$
- ◆ We can invest in information before developing the oilfield.
 - There are k alternatives of investment in information (learning options) with different costs, time-to-learn and revelation power (capacity to reduce technical uncertainty).
 - Investment in information reveals new expectations about oil reserve volume (B) and quality (q). Technical uncertainty is modeled w/ *revelation* (conditional expectation) *distributions*.
- ◆ The next slide illustrates the valuation approach with MC.

Real Options Valuation with Investment in Information

- ◆ M.C. simulation combining market (oil price) and technical uncertainties



$$F(t=0) = \text{Present Value (t=0)} = F(t=1) * \exp(-r*t)$$

$$\text{Option } F(t=1) = V - D$$

$$F(t=2) = 0$$

Expired
Worthless

Conclusions

- ◆ **Monte Carlo simulation is a very flexible tool for real options valuation of complex real life projects.**
 - We can easily combine many sources of uncertainties and include many real life restriction details and complex payoffs.
 - We discussed some MC issues like sampling (pseudo x quasi-random simulation), real x risk-neutral stochastic processes, and European x American real options problems.
 - We saw a typical example of a plant value with *input flexibility*.
- ◆ **We saw two European real option cases using MC:**
 - The Brazilian (real life) projects: the Biodiesel project and the Bolivia-Brazil gas pipeline tariff dispute.
- ◆ **We saw also a more complex (American) MC case:**
 - The learning + timing options in oilfield development.
- ◆ **Thank you very much for your time!**

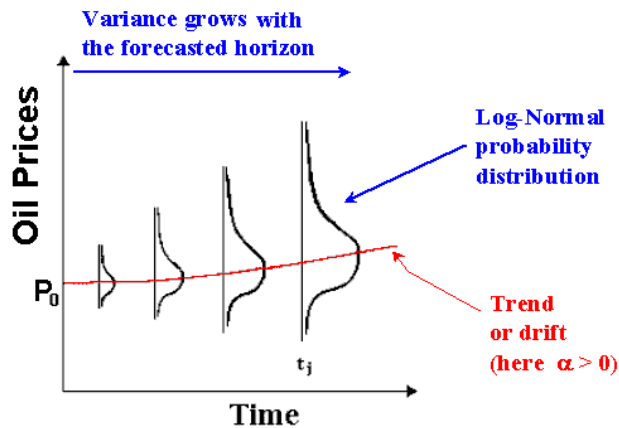
APPENDIX

SUPPORT SLIDES

Stochastic Processes for Oil Prices: GBM

◆ Like Black-Scholes-Merton equation, the classical model of Paddock et al uses the popular **Geometric Brownian Motion**

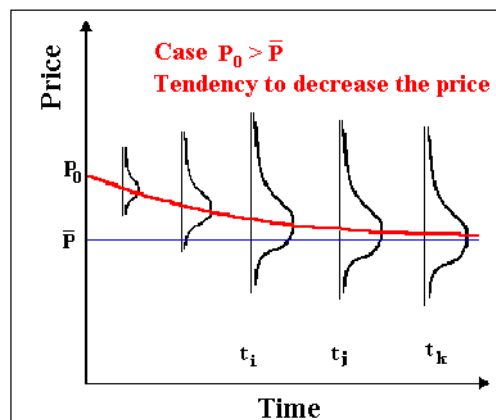
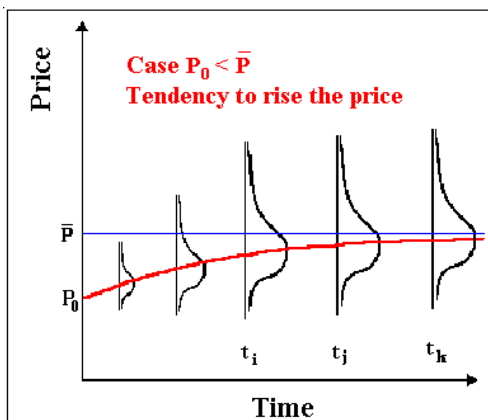
- Prices have a log-normal distribution in every future time;
- Expected curve is a exponential growth (or decline);
- The variance grows with the time horizon (unbounded)



Mean-Reverting Process

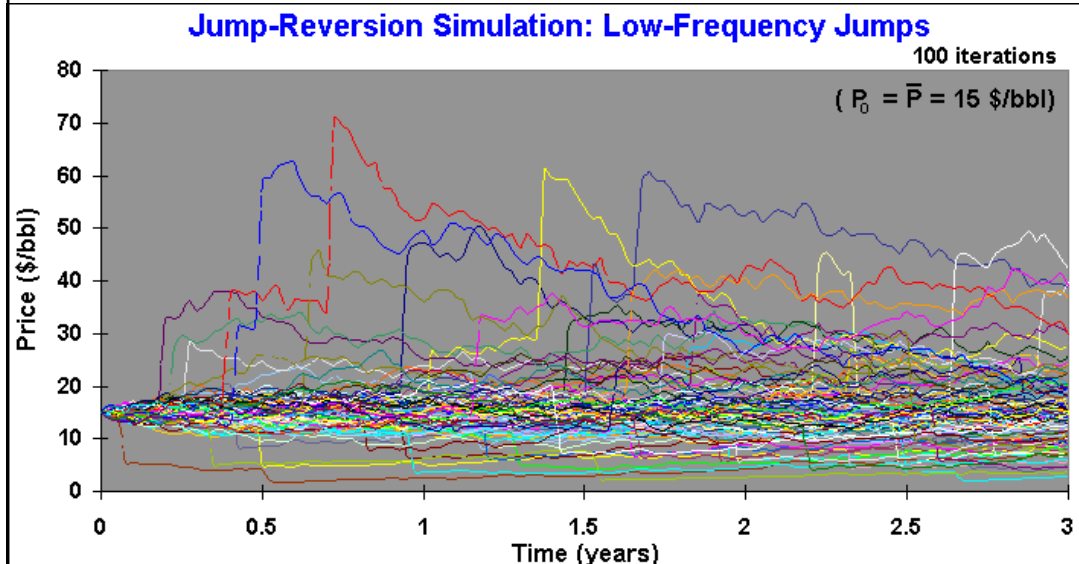
◆ In this process, the price tends to revert towards a long-run average price (or an equilibrium level) \bar{P} .

- Model analogy: *spring* (reversion force is proportional to the distance between current position and the equilibrium level).
- In this case, variance initially grows and stabilize afterwards



Mean-Reversion + Jumps: MC *Sample Paths*

- ◆ Chart shows 100 sample paths from MC simulation of mean reversion plus jumps. The starting price and the long-run equilibrium price were \$15.



VBA Code for Black-Scholes-Merton by Simulation

- ◆ Download the Excel file with unprotected VBA code, Visual Basic for Application, at: www.puc-rio.br/marco.ind/xls/qmc_black_scholes.xls

```
Option Explicit
Option Base 1
Sub QMC_BlackScholesMerton()

    Dim V As Double
    Dim K As Double
    Dim r As Double
    Dim d As Double
    Dim sig As Double
    Dim T As Double
    Dim nsim As Long
    Dim trendRN As Double
    Dim sigraiz As Double
    Dim Fsum As Double
    Dim Xj() As Double
    Dim i As Long
    Dim N01() As Double
    Dim Vt() As Double

    Randomize
    Range("start_time").Value = Now()

    V = Range("P0_")
    K = Range("D0_")
    r = Range("r_")
    d = Range("delta")
    sig = Range("sigma")
    T = Range("T_")
    nsim = Range("NSimul")

    trendRN = (r - d - ((sig * sig) / 2)) * T
    sigraiz = sig * Sqr(T)
    Fsum = 0

    ReDim Preserve Xj(1 To nsim)
    ReDim Preserve N01(1 To nsim)
    ReDim Preserve Vt(1 To nsim)
    For i = 1 To nsim
        Xj(i) = CorputBase2(i)
        N01(i) = Moro_NormSInv(Xj(i))
        Vt(i) = V * Exp(trendRN + (sigraiz * N01(i)))
        If Vt(i) >= K Then
            Fsum = Fsum + (Vt(i) - K)
        Else
            End If
    Next i
    QMC_BSM = (Fsum / nsim) * Exp(-r * T)
    Application.StatusBar = False
    Range("RealOpt").Select
    ActiveCell.FormulaR1C1 = QMC_BSM
    Range("Finish_time").Value = Now()
End Sub
```

- ◆ This function call other functions: the generation of quasi-random numbers and Normal inversion functions (next slide).

VBA Code for Black-Scholes-Merton by Simulation

- ◆ The codes below are necessary complements: generation of quasi random numbers (CorputBase2) and Normal inversion (Moro).

```
Function CorputBase2(N As Long) As Double
' Returns the equivalent first
' van der Corput sequence number
Dim c As Double, ib As Double
Dim i As Long, n1 As Long, n2 As Long
n1 = N
c = 0
ib = 1 / 2
Do While n1 > 0
    n2 = Int(n1 / 2)
    i = n1 - n2 * 2
    c = c + ib * i
    ib = ib / 2
    n1 = n2
Loop
CorputBase2 = c
End Function

Function Moro_NormSInv(u As Double) As Double
' Calculates the Normal Standard numbers given u,
' the associated uniform number (0, 1)
' VBA version of the Moro's (1995) code in C
' Option Base 1 is necessary to be declared before
' this function for vector elements positioning to work
Dim c1, c2, c3, c4, c5, c6, c7, c8, c9
Dim X As Double
Dim r As Double
Dim a As Variant
Dim b As Variant
a = Array(2.50662823884, -18.61500062529, 41.39119773534, -25.44106049637)
b = Array(-8.4735109309, 23.08336743743, -21.06224101826, 3.13082909833)
c1 = 0.337475482272615
c2 = 0.976169019091719
c3 = 0.160797971491821
c4 = 2.76438810333863E-02
c5 = 3.8405729373609E-03
c6 = 3.951896511919E-04
c7 = 3.21767881768E-05
c8 = 2.888167364E-07
c9 = 3.960315187E-07
X = u - 0.5
If Abs(X) < 0.42 Then
    r = X ^ 2
    r = X * (((a(4) * r + a(3)) * r + a(2)) * r + a(1)) / _
    (((b(4) * r + b(3)) * r + b(2)) * r + b(1)) * r + 1)
Else
    If X > 0 Then r = Log(-Log(1 - u))
    If X <= 0 Then r = Log(-Log(u))
    r = c1 + r * (c2 + r * (c3 + r * (c4 + r * (c5 + r * _
    (c6 + r * (c7 + r * (c8 + r * c9))))))
    If X <= 0 Then r = -r
End If
Moro_NormSInv = r
End Function
```

More on Simulation of Stochastic Processes & Quasi-Monte Carlo

- ◆ See equations, discussion and Excel spreadsheets on **stochastic processes simulation** at:

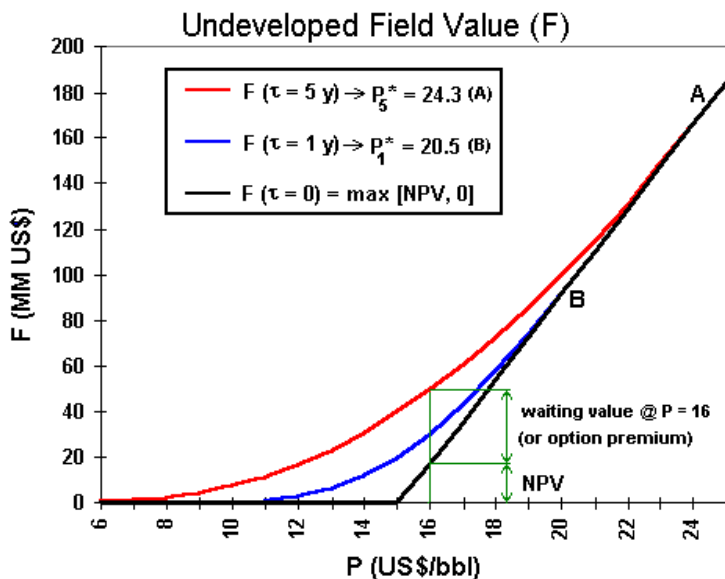
www.puc-rio.br/marco.ind/sim_stoc_proc.html

- ◆ See equations, discussion and Excel spreadsheets on **quasi-random numbers** (quasi-MC simulation) at:

www.puc-rio.br/marco.ind/quasi_mc.html

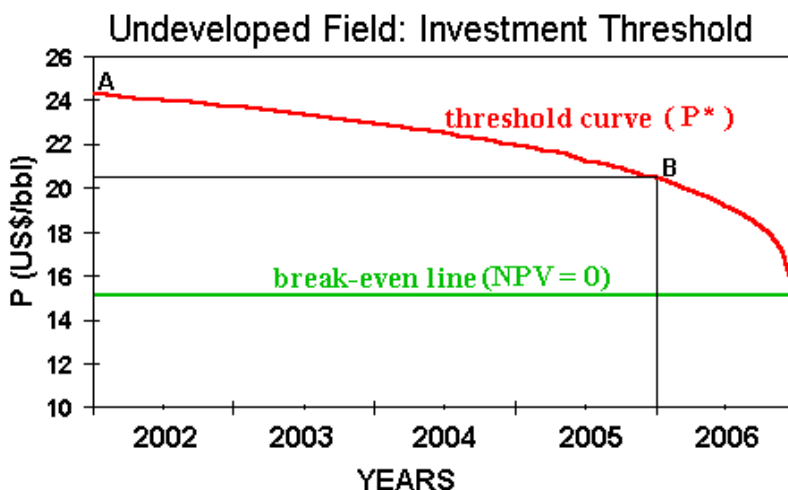
The Undeveloped Oilfield Value: Real Options and NPV

- ◆ Assume that $V = q B P$, so that we can use chart $F \times V$ or $F \times P$
- ◆ Suppose the development break-even (NPV = 0) occurs at US\$15/bbl



Threshold Curve: The Optimal Decision Rule

- ◆ At or above the threshold line, is optimal the immediate development. Below the line: “wait, learn and see”.
 - Compare the points “A” and “B” with the previous slide.



Biodiesel Business Format

- ◆ The biodiesel business format suggested by real options analysis is to enter also in the vegetable raw oil market, by allowing an excess crushing capacity (~small investment) so that we can make biodiesel and vegetable oil to market.
- ◆ In this way we have two complementary business with a real options *natural hedging* for vegetable oil prices:
 - ① The biodiesel business, where the vegetable raw oil is *cost* to transesterification (so, a *cheap* raw oil benefits this business); and
 - ② The vegetable oil to market business, where the vegetable raw oil is *revenue* (so, an *expensive* raw oil benefits this business).
- ◆ In this format, the vegetable oil is demanded either by biodiesel business or other market (e.g., food).
 - It is good for everybody: for the farmers, with grain demand either for biodiesel or for other vegetable oil market; and for Petrobras, capturing the options value from the volatile market.

The Value of Anticipating Oilfield Development

- ◆ This is based in real life case: a large oil company has a *portfolio of deep-water exploration* assets in a certain area.
 - There is more than 80% chances of at least one success.
 - ➔ The oilfield chance factor (oil existence uncertainty) is modeled with Bernoulli distributions, including correlations when relevant.
 - In case of success, we'll start the appraisal phase in order to reveal the reserve volume (B) and quality (q).
 - ➔ Technical uncertainty over B and q is modeled w/ conditional expectation distributions (conditioning is the information revealed by appraisal wells)
 - In addition, oil prices are uncertain and follows a mean-reversion plus jumps process.
- ◆ The standard investment process is:
 - Drill the exploratory prospects (~ one year);
 - Drill appraisal wells in case of success (more ~ one year); and
 - After completed the appraisal phase, develop the oilfield (if it is deep-in-the-money).

The Value of Anticipating Oilfield Development

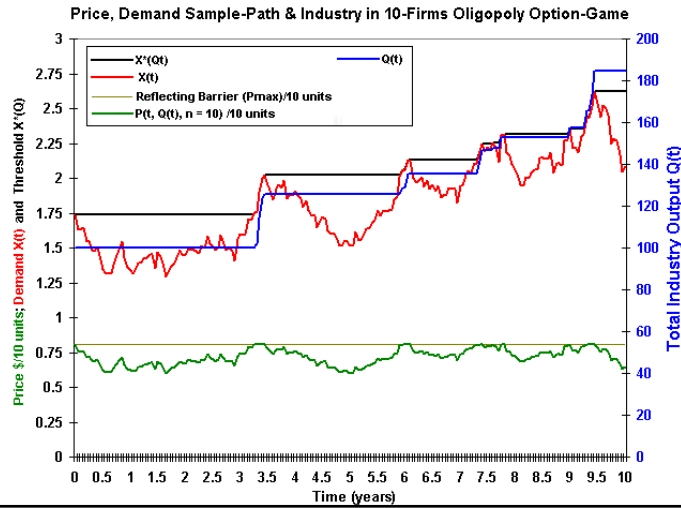
- ◆ After the development decision, the critical path to start oilfield production is the *floating production unit* (FPU).
- ◆ What if we anticipate the FPU investment, even before the exploratory drilling, in order to anticipate the production (from 1 to 2 years) of the *best* discovered oilfield?
 - We have the *option to anticipate production of the best of n risky assets* (exploratory prospects). This is a kind of *rainbow option*.
 - ➔ We also include one “insurance” asset: one already discovered oilfield (but not *deep-in-the-money*). If all the prospects are dry-holes, we can use the contracted FPU on this oilfield (but FPU will be super-dimensioned).
 - We use MC simulation (drilling success; volume and quality; oil prices) in order to quantify the two investment strategy values:
 - ➔ Traditional strategy (investing in a *taylor-made* FPU but only after the appraisal phase) x anticipating FPU strategy (with a flexible plant).
 - For the specific portfolio considered, MC simulation showed that the riskier strategy (earlier FPU investment) is more valuable.

Oligopoly under Uncertainty

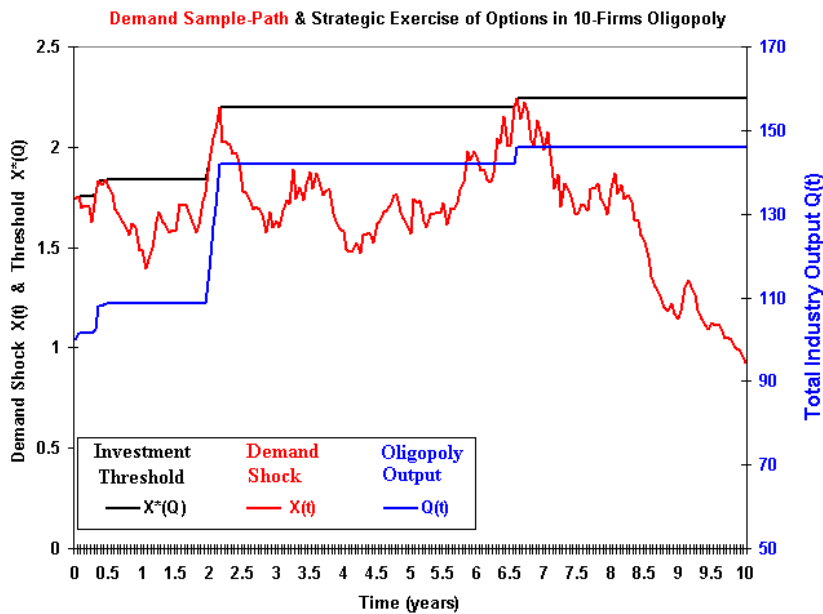
- ◆ Consider an oligopolistic industry with n equal firms
 - Each firm holds compound perpetual American call options to expand the production.
 - The output price $P(t)$ is given by a demand curve $D[X(t), Q(t)]$. Demand follows a geometric Brownian motion.
- ◆ This Grenadier’s model has two main contributions to the literature:
 - Extension of the Leahy’s “principle of optimality of myopic behavior” to oligopoly (myopic firm ignoring the competition makes optimal decision);
 - The determination of oligopoly exercise strategies using an “artificial” perfectly competitive industry with a *modified demand function*.
 - ➔ Both insights simplify the option-game solution because “*the exercise game can be solved as a single agent’s optimization problem*” and we can apply the usual real options tools, avoiding complex equilibrium analysis.
- ◆ We will see some simulations with this model in order to compare the oligopolies with few firms ($n = 2$) and many firms ($n = 10$) in term of investment/industry output levels
 - We will see the maximum oligopoly price, an *upper reflecting barrier*

Simulation in Real Options Game: Oligopoly

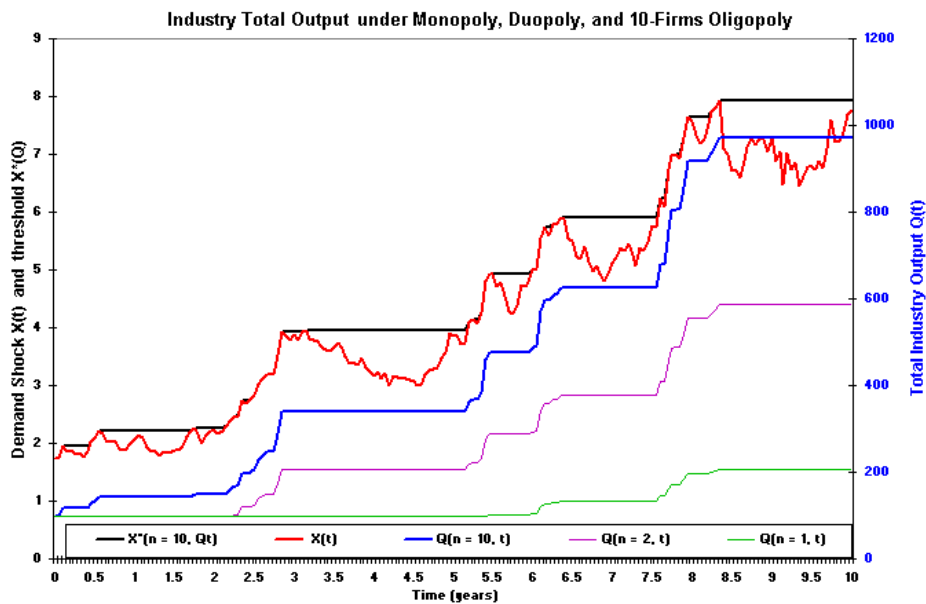
- ◆ Grenadier (2002) is a nice example of using MC to analyze an oligopoly under uncertainty using real options + game theory.
 - Simulating demand, we see the effect on industry investment option exercise, production and price (figure below shows one sample-path).



Oligopoly under Uncertainty



Oligopoly under Uncertainty



Flex Fuel Plant: Input Values

◆ In the numerical flex-fuel example, as presented in the chart, were used:

- Volatilities of 25% p.a. (for both, oil and coal)
- Reversion slowness: half-life of 3 years (for both, oil and coal); and
- Interest rate of 6% p.a.

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Quasi-Monte Carlo Numbers: Filling in the Gaps

- ◆ QMC sequence of numbers has the filling in the gaps property: next number in the sequence is placed in the largest gap.

